An Analytical Model for Lifetime of Hierarchical Wireless Ad Hoc and Sensor Networks

JERZY MARTYNA

“Institute of Computer Science Jagiellonian University
ul. Prof. S. Łojasiewicza 6, 30-348 Cracow, Poland
martyna@softlab.ii.uj.edu.pl

Received date1. Revised date2. Accepted date3.

Abstract: This paper presents an analytical model for lifetime of hierarchical wireless ad hoc and sensor networks based on energy by considering the relationship between individual nodes, cluster of nodes and the whole network. The model uses a set of hardware parameters such as data-rate, minimum required transmit power, etc. to compute a node’s average lifetime in these networks. Through comparison of the lifetime model of hierarchical and flat topologies for ad hoc and sensor networks we shown how these models can be applied to drive architectural decisions, compute energy budgets, etc. This allows designers of ad hoc and sensor networks to focus on factors that have the greatest potential impact on network lifetime.

Keywords: wireless ad hoc networks, wireless sensor networks, model of lifetime

1. Introduction

Wireless ad hoc networks [1, 2] are the ultimate frontier in wireless communication. This technology allows network nodes to communicate directly to each other using wireless transceivers without the need for a fixed infrastructure. This is a very distinguishing feature of ad hoc networks with respect to more traditional wireless, such as cellular networks and wireless LANs, in which nodes communicate with each other through base station.

Wireless sensor networks (WSNs) [3] are a particular type of ad hoc network, in which the nodes are small sensors with limited hardware capabilities. They communicate wirelessly with each other. Because all nodes are placed in a random matter, they are capable of communicating with each other within a certain communication radius.

The lifetime of a wireless sensor network as well as an ad hoc network is defined as the time after which the first node (link) disconnects [4, 5]. This definition is especially
useful for determining the lifetime of these networks functioning in real-time applications. It is also referred to as the worst-case lifetime model. In this paper, following [4, 5], we consider the time to the first node failure as the network lifetime.

The lifetime model of sensor networks and ad hoc networks was developed in study by Bhardwaj [6], and in a number of papers [4, 7, 8, 9, 10]. The upper limit of network lifetime when the data source and data rate are known was derived by Bhardwaj [6]. A distributed procedure to find such capacity achieving routes was proposed in [4]. In other papers [7, 8] energy aware routing schemes were considered. Recently, minimizing the transmission cost [9] and the placement of nodes for energy efficiency [10] have been studied. However, none of the above given models provide an analysis of lifetime in hierarchical sensor networks achieved by clustering.

The main goal of this paper is to introduce a new framework for modeling the lifetime of WSNs and ad hoc networks. The first goal of this paper is to build a lifetime model of a single sensor node (SN), a single clusterhead, a cluster, and a whole WSN. The second goal of this paper is to answer the following question: is the lifetime of a WSN better for a flat WSN or two-tier hierarchy in WSN? And also, how are the lifetimes of a sensor node and a cluster in a WSN as a function of connectivity?

In the next section we discuss the lifetime of an SN. In Section 3, we introduce a framework for achieving a lifetime of sensors clusters, clusterhead sensors and a whole WSN. We give some illustrations of our methodology in Section 4. We conclude in Section 5.

2. The Lifetime of a Sensor Node

In this section, we provide a definition of the lifetime of a sensor node (SN) in WSNs. Since sensor networks are a subclass of wireless ad hoc networks, this lifetime model applies in this type of networks also.

We have made some assumptions in our framework:

1) All the technical parameters (such as initial energy power stored in the battery, the radio transmitting range, RAM memory, etc.) are the same for all nodes in WSN,
2) The radio transmitting range of SNs is limited to their neighbors,
3) All SNs have a fixed location in the sensor field. The sink is immovable,
4) SNs can be located by physical devices or topology discovery algorithms,
5) Communications between sensors run in a two-way direction within a communication range due to limited energy power.

We assume that each node in the WSN has finite energy $E^{\text{init}}_{\text{bat}}$. The battery energy is consumed in receiving and sending messages. When the battery energy is less or
equal to $E_{\text{bat}}^*$, the node can be considered as "dead". Thus, based on the above given assumptions, it is possible to determine the lifetime of a single node, namely.

**Definition 1**

The lifetime of a continuously active SN is equal to

$$\tau_{\text{life}} = \frac{E_{\text{bat}}}{P_1}$$  \hspace{1cm} (1)

where $P_1$ is the transmit power, $E_{\text{bat}}$ is the current energy power stored in the battery. For a given spatial energy density in the network $\rho_S$, and $P_1$ we can give the maximum acceptable data-rate, $R_b^{\text{max}}$, namely

$$R_b^{\text{max}} = \frac{P_1 \cdot \rho_S}{\rho_{\text{energy}}}$$  \hspace{1cm} (2)

where $\rho_{\text{energy}}$ is the minimum spatial energy density required for full connectivity. Thus, the minimum required transmit power for full connectivity at a given data-rate $R_b$ can be written as

$$P_1^{\text{min}} = \frac{\rho_{\text{energy}} \cdot R_b}{\rho_S}$$  \hspace{1cm} (3)

The above relationship indicates that for a fixed number of nodes, if the nodes are damaged during the elapsed time, the minimum transmitting power must increase proportionally to preserve full connectivity in the network. This relationship also implies that when the node spatial density decreases and the minimum required transmit power $P_1$ is constant, then the data-rate $R_b$ must to be reduced.

Additionally, if $R_b$ and $P_1$ are fixed, the critical minimum node spatial density for full connectivity in WSN can be written as

$$\rho_S^{\text{min}} = \frac{\rho_{\text{energy}} \cdot R_b}{P_1}$$  \hspace{1cm} (4)

Now, we can provide a better definition of the lifetime of a SN in a WSN.

**Definition 2**

When the data-rate $R_b$ and spatial energy density $\rho_S$ in the WSN are fixed, the lifetime of a node in the WSN can be formulated as

$$\tau_{\text{life}} = \frac{E_{\text{bat}} \cdot \rho_S}{R_b \cdot \rho_{\text{energy}}}$$  \hspace{1cm} (5)

or

$$\tau_{\text{life}} = \frac{(E_{\text{bat}}^{\text{init}} - t \cdot \rho_{\text{cons}}) \rho_S}{R_b \cdot \rho_{\text{energy}}}$$  \hspace{1cm} (6)
where $E_{\text{init}}$ is the initial energy power stored in the battery, $e_{\text{cons}}$ is the rate of energy consumed by a sensor, and $t$ is the elapsed time.

Note that a reduction in the data-rate will increase the lifetime of a continuously active sensor and thus the lifetime of a whole WSN.

After transforming the last equation, we can obtain the relationship for the normalized lifetime of SN, NLS. The normalized lifetime of an SN is defined as the normalized remaining energy of the sensor at the moment $t$, namely

$$NLS = \frac{E_{\text{init}}}{E_{\text{init}}} \frac{e_{\text{life}}}{e_{\text{bat}}} = \frac{(1 - t \cdot e_{\text{rel}})}{R_b \cdot \rho_{\text{energy}} S}$$

(7)

where $e_{\text{rel}}$ is the relative energy consumption of a sensor node and is equal to the ratio $e_{\text{cons}} / e_{\text{bat}}$. Thus shows the relative speed of the energy consumed by a SN.

To calculate the energy consumption of each message transmission, we borrow the energy model used in [11]. The energy consumed when the sensor receives a message of size $k$ is given by

$$\epsilon_{\text{rec}} = \epsilon_{\text{elec}} \cdot k$$

(8)

where $\epsilon_{\text{elec}}$ is the energy consumed per one received bit. The energy consumed on sending a message of size $k$ is given by

$$\epsilon_{\text{send}} = \epsilon_{\text{elec}} \cdot k + \epsilon_{\text{amp}} \cdot r^2 \cdot k$$

(9)

where $\epsilon_{\text{elec}}$ is the consumed energy per one sent bit, $\epsilon_{\text{amp}}$ is the consumed energy by an amplifier, $r$ is the radio transmitting range of a single SN.

In this way, we obtain two amounts of energy consumed in one query message at the moment $t$, namely

$$\epsilon_q^{(t)} = \epsilon_q^{(t)}_{\text{rec}} + \epsilon_q^{(t)}_{\text{send}}$$

$$\epsilon_r^{(t)} = \epsilon_r^{(t)}_{\text{rec}} + \epsilon_r^{(t)}_{\text{send}}$$

(10)

(11)

where $\epsilon_q^{(t)}$ is the energy consumed in receiving, and $\epsilon_q^{(t)}$ is the energy consumed in sending. When we assume that all query messages are the same, $\epsilon_q^{(t)}$ and $\epsilon_r^{(t)}$ can be reduced to $\epsilon_q^{(t)}$ ($\epsilon_r^{(t)}$).

According to the results given in a paper by Chen [7], the energy consumption ratio when a SN is in idle mode, receiving mode and sending mode is: $1 : 1.2 : 1.68$. Therefore, the sending mode is the most exhaustive in terms of power consumption.

We can also determine the lifetime of a continuously active SN in more simpler form. We assume that for a given data-rate $R_b$ the time taken to transmit one packet (message)
is $L/R_b$, where $L$ is the mean packet length. The total amount of energy consumed per transmitted packet (message) can be given by

$$E_{\text{packet}} = P_1 \cdot \frac{L}{R_b} \text{ (dimension [J])}$$ (12)

Let the transmission of a packet with an average rate be $\lambda_p$ and the average energy consumed per second $\lambda_p \cdot E_{\text{packet}}$. Thus, the total time required to completely exhaust the initial battery energy and thereby the lifetime of a SN is given by

$$\tau_{\text{sensor life}} = \frac{E_{\text{bat init}}}{\lambda E_{\text{packet}}} = \frac{E_{\text{bat init}} R_b}{\lambda P_1 L} \text{ (dimension [s])}$$ (13)

We now point out that the above results are obtained by using a uniform traffic assumption (i.e. all nodes generate the same amount of traffic load) and all nodes consume the same energy per packet whether an SN is receiving or sending packets.

3. Determining the lifetime of clusterhead, cluster and fully hierarchically organized WSN and ad hoc networks

In this section, we will determine the lifetime of hierarchical organized WSNs and ad hoc networks.

WSN and ad hoc networks are often divided into groups defined as clusters. This concept was introduced by Baker et al. [13]. Such an approach allows us to manage a cluster and relay the collected data. In a paper by Ramamoorthy et al. [14] an expanding ring approach was used in which the cluster depth is progressively relaxed until the desired cluster size is exceeded. A hierarchy of networks is achieved by clustering technique. The first level of this hierarchy consists of SNs which "control" other SNs. They are often referred to as clusterheads. These nodes are natural places to aggregate and compress traffic data from many sensors. The communication between the clusterheads is dominating and is treated as backbone transmission. This communication is suitable for data transmission to the sink of a WSN.

In hierarchical WSNs and ad hoc networks all SNs are assigned to one of the clusters. There can be two ways of building clusters in a WSN. In the first option each cluster must be at most two hops away from any other node. In the second, one-hop clusters are considered. Since the battery of clusterheads will tend to be exhausted more quickly, it is desirable that all SNs in a cluster have equal battery capacities at any point. Therefore, the clustering algorithm should be able to rotate the clusterheads. If there is a lack of rotating clusterheads, we propose here the following definition.
Definition 3
The lifetime of a clusterhead in the hierarchical ad hoc/WSN in case of lack of rotating clusterheads is given by

$$\tau_{\text{clshead\,life}} = \frac{(E_{\text{bat}}^{\text{init}} - t \cdot e_{\text{rel}}^{\text{cons}}) \rho_S}{R_b \cdot \rho_{\text{energy}}^{\text{min}} \cdot (c_1 \cdot r^\alpha + c_2 \cdot n_s)}$$  \hspace{1cm} (14)$$

where $r$ is the coverage radius of a clusterhead, $n_s$ is the number of cluster members, $\alpha$ is the path-loss coefficient, and $c_1$ and $c_2$ are constants.

It can be seen that the maximization the hierarchical network lifetime involves maximizing the lifetime of all clusterheads or maximizing the minimum lifetime over all clusterheads.

Now we can formulate determine the lifetime of a cluster in hierarchical WSN/ad hoc networks, namely

Definition 4
The lifetime of a cluster in an hierarchical ad hoc network/WSN can be written as

$$\tau_{\text{cluster\,life}} = \frac{1}{N_c} \sum_{i=1}^{N_c} w_i^{(1)} \cdot \tau_{\text{sens\,life},i}$$  \hspace{1cm} (15)$$

where $N_c$ is the total number of SNs at the moment when the network starts and $w_i^{(1)}$ is the weight of each node stated as follows

$$w_i^{(1)} = \frac{c_3}{d_i}$$  \hspace{1cm} (16)$$

where $d_i$ is the distance from the $i$-th node to the clusterhead of a cluster.

Naturally, for both of the above given lifetimes we can form the normalized lifetimes of a clusterhead and a cluster.

Before we formulate the lifetime definition an entire hierarchical ad hoc and WSN network, we recall that there is is some difference between both of them. An ad hoc network is built to transport a data. On the other hand, a WSN is not designed to transport data, but rather to observe a region. Therefore, the WSN should be able to fulfill its duty for as long as possible.

There are a number of possible various definitions of an WSN lifetime.

1) The time until the first node fails, which is used in Real-Time WSNs,

2) The time until 50% of the nodes run out of energy and become inactive. Any other fixed percentile is applicable as well,

3) The time until there is a spot in an WSN that is not covered by the network,
4) The time until a network position (when there are two nodes in that a WSN can no longer communicate with each other) [15].

The need to maximize the lifetime of a WSN improves network performance. It means, that each of the given lifetime definitions requires different solutions. Nevertheless, we propose here two suitable definitions of flat and hierachical WSN and ad hoc network lifetimes.

For a flat WSN and ad hoc network (i.e. networks without a two- or multi-level hierarchy), we can give the following definition.

**Definition 5**

The lifetime of a flat ad hoc/WSN network is given by

\[
\tau_{\text{network}}^{\text{life}} = \frac{1}{N} \sum_{i=1}^{N} w_i^{(2)} \cdot \tau_{\text{sensor}}^{\text{life},i}
\]  

where \( N \) is the total number of SNs in a WSN at the moment \( t = 0 \) and \( w_i^{(2)} \) is the weight of each node, namely

\[
w_i^{(2)} = \frac{c_4}{d_i^2}
\]  

where \( d_i \) is the distance from the \( i \)-th node to the sink of the network.

The above definitions not consider a start up of a spot of nodes without transmitting range, as well as a possible a lack between two neighboring nodes.

For the two- (or multi) level network hierarchy, we can formulate the following definition.

**Definition 6**

The lifetime of a two-layer hierarchical WSN/ad hoc network is given

\[
\tau_{\text{network}}^{\text{life}} = \frac{1}{K} \sum_{k=1}^{K} w_k^{(3)} \cdot \tau_{\text{clusterhead}}^{\text{life},k}
\]  

where \( K \) is the total number of clusterhead nodes, \( w_k^{(3)} \) is the weight of each clusterhead node given as as follows

\[
w_k^{(3)} = \frac{c_5}{d_k^2}
\]  

where \( d_k \) is the distance from clusterhead \( k \) to the sink of a WSN.
4. Numerical Experiments

In this section, we give some numerical results of lifetime modelling in hierarchical ad hoc and sensor networks.

We assumed that in our model of ad hoc and WSN the node spatial density of a network can change over time. Among other things, SNs are lost, when the batteries are exhausted. A WSN changes its node spatial density. We define the initial density of SNs when the network starts as $N_0$. Keeping the sensor field constant, the node spatial density is decreased if the number of SNs becomes lower than $N_0$. The change in the number of SNs is defined as $\Delta N = N_0 - N_f$, where $N_f$ is the final number of SNs.

![Fig. 1. Lifetime of a node as function of transmit power](image1)

![Fig. 2. Lifetime of a cluster as function of transmit power](image2)
By using the Eq. (13) we obtained the lifetime of an SN as a function of transmit power $P_1$ for a given $\Delta N = 100, 200$ (see Fig. 1). It is evident that the node lifetime decreases as the transmit power increases to support communication in a sparser network.

Considering the lifetime of a clusterhead as a function of the transmit power, the behavior of the lifetime of a clusterhead is plotted in Fig. 2. It can be observed that the lifetime of a clusterhead is less than the lifetime of an ordinary sensor.

The same effect is observed for the lifetime of a hierarchical ad hoc or WSN network (see Fig. 3). This suggests that a long of lifetime is possible when the level of connectivity is small. Thus, the spatial density is low.

![Fig. 3. Lifetime of a hierarchical ad hoc/WSN as a function of transmit power](image)

5. Conclusion

In this paper we presented a lifetime model for hierarchical sensor and ad hoc networks. This model can be a very powerful analytic tool in WSN design as it can be used to derive many performance parameters of interest. Among other things, it helps showing which organization of an WSN or ad hoc network is better: a flat structure or two-level (multi-level) hierarchy? In general, complete lifetime modeling framework provides an analytic tool for assessing energy consumption in these networks.

Our future work involves using this model as an analysis tool for some routing algorithms in WSNs, as well as damages and mobility in these wireless networks.
References

Model analityczny czasu życia hierarchicznych, bezprzewodowych sieci sensorowych oraz ad hoc

Streszczenie

W pracy przedstawiono model analityczny czasu życia hierarchicznych, bezprzewodowych sieci sensorowych i ad hoc. Model ten bierze pod uwagę wzajemne zależności pomiędzy poszczególnymi węzłami, klastrami oraz całą siécią. Uwzględniono w nim sprzętowe parametry sieci, takie jak przestrzenna gęstość rozmieszczenia węzłów, minimalna energia transmisji danych, intensywność transmisji. Przedstawione modele umożliwiają porównanie ze sobą płaskich i hierarchicznych topologii sieci ad hoc oraz sieci sensorowych, co jest niezbędne w ich projektowaniu. Pozwala to na znalezienie takiej topologii sieci, której czas życia jest najdłuższy.