Using singular value decomposition in textile production quality control

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Abstract: The article presents a possibility of using singular value decomposition in textile quality control. This approach consists in using only first singular values in the analysis and examining their standard deviation from the referential values. The examined object is textile, represented in the form of an image and saved as a data matrix. The article includes comparison of the efficiency and speed of different data analysis algorithms and methods. SVD method, in regard to defect detection, shows a high efficiency. Furthermore, the speed of the proposed solution is comparable with the fastest algorithms and is the best from among the methods with the same efficiency. The results of the conducted and described experiment consisting in examining five different textile defects confirm the potential of the method chosen.

Keywords: Textile quality control, singular value decomposition, SVD

1. Introduction

Quality control in today’s production systems is as important, as the control process itself. These elements often work together in a feedback system. Indeed, this solution has been applied for a long time, but nowadays, in quality control systems computers and complex data analysis algorithms have been widely used. It stems mainly from the quality examination of more and more complex objects or processes. Depending on the type of the examined object, this complexity may be so high that the measurement consists in the analysis of data saved as images obtained from vision systems. Nowadays, we use analyses of various data, whose images are textures and these are e.g.: paper, textile, wood, pottery, flames, clouds, astronomical objects, DNA code and many others. Depending on the objects analysed, various methods of image processing are applied. From the simplest ones, statistic or stochastic, through methods based on transforms and decompositions, to the most advanced methods using artificial intelligence. The
former ones are characterised by high speed, but they have low efficiency. Methods based on transforms or decomposition are efficient, but because of their complex numeric algorithms, they are too slow. Methods based on artificial intelligence were to bring a satisfactory efficiency and sufficient speed. These methods, however, require learning, which, depending on the complexity of the object takes time. And vice versa, the time of learning influences efficiency, so those methods gain a satisfactory efficiency only after some time. The following article proposes a solution characterised by high efficiency and relatively short processing time. The chosen algorithm is considered to be one of the most efficient methods, based on data decomposition. Reduction of processing time was obtained using an SVD decomposition feature, consisting in using only the first singular value. This procedure results in loss of efficiency, but because of the properties of SVD decomposition, this loss is slight.

2. Singular Value Decomposition (SVD)

Singular value decomposition consists in saving one data set as another data set, preserving most of the information about this set. For each real matrix A with the dimensions m x n, where m is the number of rows and n is the number of columns, there are such orthogonal U matrices with the dimensions m by m and V with the dimensions n by n, for which the following dependence is true:

\[ U^T AV = \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_l) \]

where \( l = \min(m, n) \).

If the size of A matrix is n, then: \( \sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0, \sigma_{r+1} = \sigma_{r+2} = \ldots = \sigma_l = 0 \)

U and V matrices are orthonormal, i.e. orthogonal \( U^{-1} = U^T \) and \( V^{-1} = V^T \) and the length of each vector equals 1. Those matrices are called right and left singular matrices respectively.

Matrix \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_n) \) is a diagonal matrix of singular values \( \sigma \) with the dimensions m x n set in a descending order. The first singular value \( \sigma_1 \) located in the Sigma matrix in \( \Sigma_{11} \) position has the greatest importance and highest sensitivity for any disturbances in the analysed data [4].

There are many algorithms of numeric singular value matrix decomposition. One of them, proposed by G.H. Golub and C. Reinsch is a numerically stable algorithm, based on orthogonal transformation of the input matrix using Householder transform to a bidiagonal matrix, which is subject to diagonalization, resulting in obtaining singular values. [6].

The basic feature of SVD is that by using only the first singular values, it is possible to reconstruct the original data preserving their most relevant features using the dependence 2:
\[ A = USV^T \] (2)

With such limitation, the information content, entropy and energy of the reconstructed image still remain on a very high level. Using this most important feature of singular value decomposition, the possibilities of the method in defect detection according to its efficiency and speed of analysis of textile images was examined.

**Other methods of image analysis**

There are many methods of quality image analysis, so there are also many ways of their classification. It seems reasonable to divide texture analysis methods according to image processing into:

- statistical, stochastic
  - histogram examination [24]
  - LBP – local binary pattern [7] [18]
  - GLCM - co-occurrence - event matrix method [19] [9]
- transforms
  - Gabor filters [23] [24]
  - DCT – discrete cosine transform [8]
  - DWT – discrete wavelet transform [3]
- decompositions
  - PCA – eigen values analysis [11] [20].
  - KLT – Karhunen-Loeve transform [22]
  - SVD – singular values decomposition [4]
- methods based on artificial intelligence [13] [14]

Since the 70s, those methods have been constantly developed and combined, thus making the image analysis more and more effective [10].

**Comparative study of the efficiency of image analysis methods**

Those methods may be used in qualitative image analysis, we just have to choose the right classifier, which will make a classification according to predefined quality thresholds. Some of these methods are very efficient, the others very quick. It is difficult to
find a method that would be both very efficient and very fast. There are many publications comparing efficiency and speed of algorithms [17]. The author of one paper observed that the best efficiency is obtained by the image analysis methods based on decompositions, PCA and KLT, obtaining a result over 85%.

**Comparison of the speed of image analysis methods**

To compare SVD algorithm speed to other decomposition-based algorithms, comparative analysis in Matlab software was conducted. The same methods, analysed before in regard to their efficiency, were chosen for the study. The speed of algorithms was compared for images in 8-bit colour mode and the size of $512 \times 512$ and $1024 \times 1024$ pixels in the same hardware and software conditions: computer AMD Sempron 2800 MHz 1GB RAM, operating system Windows 2000. The results are shown in table 1. Because of large differences in computational time, a logarithm scale on a time axis was used to visualise the results. It enables us to show the relationship between the speed of specific algorithms clearly.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Method</th>
<th>Time processing, ms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matrix size</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$512 \times 512$</td>
<td>$1024 \times 1024$</td>
</tr>
<tr>
<td>Statistical</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>histogram</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>LBP</td>
<td>1970</td>
</tr>
<tr>
<td></td>
<td>GLCM</td>
<td>94</td>
</tr>
<tr>
<td>Transforms</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>DCT</td>
<td>140</td>
</tr>
<tr>
<td></td>
<td>DWT</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>DWHT</td>
<td>15</td>
</tr>
<tr>
<td>Decompositions</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>PCA</td>
<td>12440</td>
</tr>
<tr>
<td></td>
<td>KLT</td>
<td>2750</td>
</tr>
<tr>
<td></td>
<td>SV16</td>
<td>410</td>
</tr>
</tbody>
</table>

Tab. 1. Comparison of the processing time of various image analysis algorithms
On the basis of the measurements conducted we may say that among from the compared methods, those characterised by high efficiency are the slowest. In the picture 2 it was shown that singular value decomposition method using only the first singular value is at least one order of magnitude faster than other decomposition-based methods, e.g.: KLT and PCA based on eigenvalues. The proposed approach is also comparable with the fastest methods based on transforms and even faster than some statistical methods.

3. Using singular value decomposition method to analyze textile images

Textile images have some common qualities, which we can use in defect analysis. These are, for example:

- Uniform textile colour with different possible hues, according to structural parameters, construction of the yarn, materials used or light reflection coefficient for specific fibres.
- Uniform surface in the same areas, resulting from the structure or workmanship.
- Identical effect on the reflected light, in which images of the same textile were taken.

Preserving identical method of observation for all the samples it is possible to analyze textiles with regard to the occurrence of differences between its separate elements.
To analyze images of large textile surface area, we propose to divide into smaller fragments and each fragment should be analyzed individually. Textile image shall be represented by a matrix of data, whose elements shall constitute values of separate pixels of this image. The complexity of the analyzed images determine whether it is enough to analyze a black-and-white or colour image with complex RGB or CMYK colour modes, the analysis of which shall be conducted similarly to black-and-white images, the difference being that we do not analyze just one component of the image, but 3 or 4, respectively.

Using one of the SVD decomposition features, which says that in a diagonal matrix $\Sigma$, singular values are arranged in descending order and the first singular value $\sigma_1$ situated on the diagonal has the largest weight and sensitivity, much larger than the following singular values, so we use only this one to analyze textile images [4]. It was therefore assumed that the first singular value is enough to preserve the information on textile defect. In the picture 2, images reconstructed using the first singular value are presented, both the fragment without defects and the one with defects. The picture 2.d shows that after image reconstruction using the first singular value, defect information is not lost.

The shape, texture, surface and colour of a textile have the largest influence on singular values in textile images. On the basis of the above, the first singular values of textile
images without defects and images of the same textiles with defects differ significantly. To use this method to textile production quality control, it is enough to determine the range of singular values for textiles without defects. In other words, it is enough to determine the maximum standard deviation of the first singular values for RGB colour mode. All the images of textile fragments, whose singular values are not included in the determined range, or for which the standard deviation is higher than the maximum deviation for images without defects should be treated as images with defects or images that do not meet quality standards.

4. The experiment results

The algorithm using SVD method to detect textile defects was designed in Matlab environment. Textile image was loaded to memory and saved in the form of a matrix. SVD matrix decomposition was made using SVD function. In the result of the decomposition, two matrices U and V and one real singular value vector S, whose first element is the basis for detecting textile defects were obtained. During the experiment, such quality classification rule was employed that the images with standard deviation of at least twice the size of the maximum permissible standard deviation for images without defects were classified as images with defects. This standard deviation may be expressed using formula 3:

\[ D = \frac{\sum_{i=1}^{n} |x_i - \bar{x}|}{n} \]  

where
- \( x_i \) – first singular values of textile image for R, G, B colour mode, respectively.
- \( n \) – number of images without defects
- \( \bar{x} \) – mean singular values expressed by formula 4:

\[ \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \]  

Under the same lightning conditions and reflected light, textile pictures were taken using a digital camera with 7,1 million pixel resolution. From each textile, five 100×100mm fragments of different parts were chosen. Textile images were colour in 24 bit RGB colour mode and were 500×500 pixels large with the resolution of 5 pixels by 1mm. Five different types of textile defects were analyzed. For each defect type, three textiles were examined. Out of each textile, five samples were separated, four without defects and one with a defect to be detected. In total, 75 textile samples, size 500 by 500 pixels, were examined. Using the above quality classification rule, for each examined
textile the piece with defects was accurately detected. Because of the large scope of the results, measurement results only for the most distinctive textile for each defect type are presented in table 2. The first three values in the “standard deviation” column are the maximum deviations from the mean singular values for RGB colour mode for textile images without defects.

<table>
<thead>
<tr>
<th>Defect</th>
<th>Images without defects, meeting quality standards</th>
<th>Image with defects</th>
</tr>
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<tbody>
<tr>
<td>bleach</td>
<td>Under each image there are first singular values for three R, G, B colour mode</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean values: R: 1.71 G: 1.51 B: 1.47</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 9.89 G: 9.89 B: 9.89</td>
<td></td>
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<tr>
<td></td>
<td>Max values: R: 0.86 G: 0.86 B: 0.86</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.62 G: 0.62 B: 0.62</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min values: R: 0.40 G: 0.40 B: 0.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.40 G: 0.40 B: 0.40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean values: R: 0.52 G: 0.52 B: 0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.52 G: 0.52 B: 0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max values: R: 0.49 G: 0.49 B: 0.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.49 G: 0.49 B: 0.49</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min values: R: 0.36 G: 0.36 B: 0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.36 G: 0.36 B: 0.36</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean values: R: 0.22 G: 0.22 B: 0.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.22 G: 0.22 B: 0.22</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max values: R: 0.19 G: 0.19 B: 0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>with no defects: R: 0.19 G: 0.19 B: 0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min values: R: 0.14 G: 0.14 B: 0.14</td>
<td></td>
</tr>
</tbody>
</table>

Tab. 1. Textile images together with the first singular values for RGB colour mode
We may observe that the standard deviations from the singular values of RGB image colour mode without defects are slight - from a few up to several dozens percent. SVD decomposition time of one sample using a computer with AMD Sempron 2800 CPU and 512MB RAM memory was 0.3 sec. With such computer technical parameters, linear velocity of the analyzed textile would be 47m²/min.

5. Summary and conclusions

On the basis of the experiment results, using quality assessment criterium based on standard deviation at least two times higher for the defect images than standard deviations for images without defects, we may say that textile quality control using singular value decomposition is possible. This algorithm efficiently distinguishes between samples without defects from samples with defects or samples that do not meet quality requirements. Satisfactory results have been obtained for images of almost regular textures, as well as irregular ones. The obtained results prove that the proposed solution, not less efficient than other decomposition-based methods, is much faster. We should also take into consideration the fact that SVD method is competitive in terms of speed with the transform-based methods and even faster than some of the statistical methods. Because defect detection is based on the analysis of the first textile image singular values only, the implementation of this method is simpler than of other static or stochastic methods or requiring e.g. using artificial intelligence. It is a significant advantage, as there is no need to prepare learning sets, which takes much time and, besides, learning algorithms have high computational complexity and, as a result, they require large quantities of computational power in case of quality control in real-time. The experiment conducted shows that such hardware configuration allows us to obtain quite high linear velocity 47m²/min, and a proper implementation of parallel singular value decomposition would enable us to control quality in real-time at much higher velocity.

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Wykorzystanie dekompozycji według wartości osobliwych w kontroli jakości produkcji tkanin

Streszczenie

Artykuł przedstawia możliwość wykorzystania dekompozycji według wartości osobliwych SVD do kontroli jakości tkanin. Podejście to polega na wykorzystaniu do analizy tylko pierwszych wartości osobliwych i badaniu ich odchylenia standardowego od wartości przyjętych za odniesienie. Badany obiekt to tkanina, przedstawiony w postaci obrazu, zapisanego jako macierz danych. Artykuł zawiera porównanie skuteczności i szybkości różnych algorytmów i metod analizy danych. Metoda SVD pod względem detekcji defektów zapowiada dużą skuteczność. Także szybkość zaproponowanego rozwiązania jest porównywalna z najszybszymi algorytmami i jest najlepsza wśród metod o tej samej skuteczności. Wyniki eksperymentu badania pięć różnych defektów tkanin potwierdzają możliwości badanej metody.